8-7

Radical Functions

TEKS 2A.9.A Quadratic and square root functions: … investigate … the effects of parameter changes on the graphs of square root functions ….

Objectives

Graph radical functions and inequalities.

Transform radical functions by changing parameters.

Vocabulary

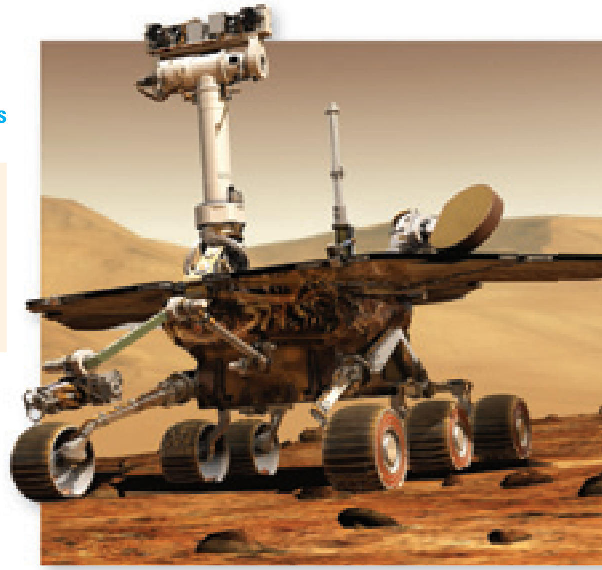
radical function

square-root function

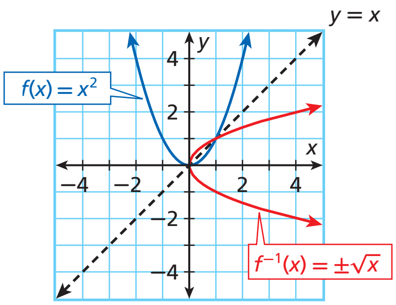
Also 2A.1.A, 2A.4.A, 2A.4.B, 2A.4.C, 2A.9.B, 2A.9.E, 2A.9.F, 2A.9.G

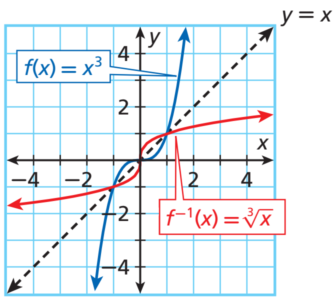
Who uses this?

Aerospace engineers use transformations of radical functions to adjust for gravitational changes on other planets. (See Example 5.)



Recall that exponential and logarithmic functions are inverse functions. Quadratic and cubic functions have inverses as well. The graphs below show the inverses of the quadratic parent function and the cubic parent function.





Notice that the inverse of $f(x) = x^2$ is not a function because it fails the vertical line test. However, if we limit the domain of $f(x) = x^2$ to $x \geq 0$, its inverse is the function $f^{-1}(x) = \sqrt{x}$.

A **radical function** is a function whose rule is a radical expression. A **square-root function** is a radical function involving $\sqrt{x}$. The square-root parent function is $f(x) = \sqrt{x}$. The cube-root parent function is $f(x) = \sqrt[3]{x}$.

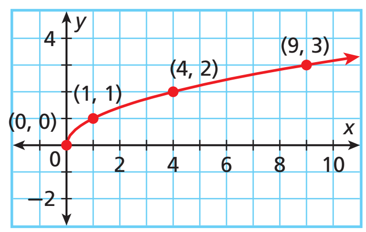
EXAMPLE 1 Graphing Radical Functions

Graph the function, and identify its domain and range.

A $f(x) = \sqrt{x}$

Make a table of values. Plot enough ordered pairs to see the shape of the curve. Because the square root of a negative number is imaginary, choose only nonnegative values for x.

x	$f(x) = \sqrt{x}$	$(x, f(x))$
0	$f(0) = \sqrt{0} = 0$	(0, 0)
1	$f(1) = \sqrt{1} = 1$	(1, 1)
4	$f(4) = \sqrt{4} = 2$	(4, 2)
9	$f(9) = \sqrt{9} = 3$	(9, 3)



The domain is $\{x \mid x \geq 0\}$, and the range is $\{y \mid y \geq 0\}$.

Helpful Hint

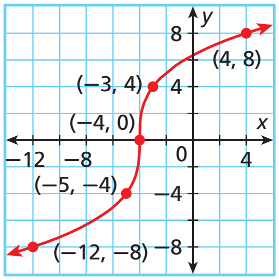
When using a table to graph square-root functions, choose x-values that make the radicands perfect squares.

Graph the function, and identify its domain and range.

B $f(x) = 4\sqrt[3]{x+4}$

Make a table of values. Plot enough ordered pairs to see the shape of the curve. Choose both negative and positive values for x.

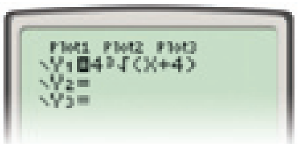
x	$4\sqrt[3]{x+4}$	$(x, f(x))$
-12	$4\sqrt[3]{-12+4} = 4\sqrt[3]{-8} = -8$	$(-12, -8)$
-5	$4\sqrt[3]{-5+4} = 4\sqrt[3]{-1} = -4$	$(-5, -4)$
-4	$4\sqrt[3]{-4+4} = 4\sqrt[3]{0} = 0$	$(-4, 0)$
-3	$4\sqrt[3]{-3+4} = 4\sqrt[3]{1} = 4$	$(-3, 4)$
4	$4\sqrt[3]{4+4} = 4\sqrt[3]{8} = 8$	$(4, 8)$

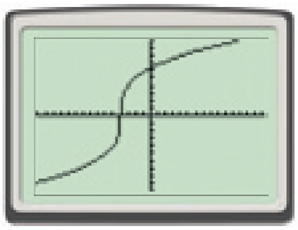


> **Helpful Hint**
>
> When using a table to graph cube-root functions, choose x-values that make the radicands perfect cubes.

The domain is the set of all real numbers. The range is also the set of all real numbers.

Check Graph the function on a graphing calculator.





The graphs appear to be identical.

CHECK IT OUT! **Graph each function, and identify its domain and range.**

1a. $f(x) = \sqrt[3]{x}$ **1b.** $f(x) = \sqrt{x+1}$

The graphs of radical functions can be transformed by using methods similar to those used to transform linear, quadratic, polynomial, and exponential functions. This lesson will focus on transformations of square-root functions.

Know it! Note

Transformations of the Square-Root Parent Function $f(x) = \sqrt{x}$

Transformation	$f(x)$ Notation	Examples
Vertical translation	$f(x) + k$	$y = \sqrt{x} + 3$ 3 units up $y = \sqrt{x} - 4$ 4 units down
Horizontal translation	$f(x - h)$	$y = \sqrt{x-2}$ 2 units right $y = \sqrt{x+1}$ 1 unit left
Vertical stretch/compression	$af(x)$	$y = 6\sqrt{x}$ vertical stretch by 6 $y = \frac{1}{2}\sqrt{x}$ vertical compression by $\frac{1}{2}$
Horizontal stretch/ compression	$f\left(\frac{1}{b}x\right)$	$y = \sqrt{\frac{1}{5}x}$ horizontal stretch by 5 $y = \sqrt{3x}$ horizontal compression by $\frac{1}{3}$
Reflection	$-f(x)$ $f(-x)$	$y = -\sqrt{x}$ across x-axis $y = \sqrt{-x}$ across y-axis

EXAMPLE 2 Transforming Square-Root Functions

Math Builders

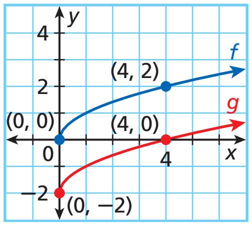
For more on transformations of functions, see the Transformation Builders on pages xxv and xxvii.

Using the graph of $f(x) = \sqrt{x}$ as a guide, describe the transformation and graph each function.

A $g(x) = \sqrt{x} - 2$

$g(x) = f(x) - 2$

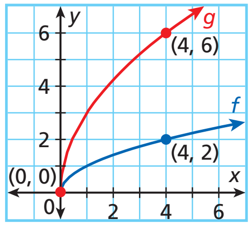
Translate f 2 units down.



B $g(x) = 3\sqrt{x}$

$g(x) = 3 \cdot f(x)$

Stretch f vertically by a factor of 3.



CHECK IT OUT! **Using the graph of $f(x) = \sqrt{x}$ as a guide, describe the transformation and graph each function.**

2a. $g(x) = \sqrt{x} + 1$ **2b.** $g(x) = \frac{1}{2}\sqrt{x}$

Transformations of square-root functions are summarized below.

Know it! Note

$|a| \longrightarrow$ vertical stretch or compression factor
$a < 0 \longrightarrow$ reflection across the x-axis

$h \longrightarrow$ horizontal translation

$$f(x) = a\sqrt{\frac{1}{b}(x - h)} + k$$

$|b| \longrightarrow$ horizontal stretch or compression factor
$b < 0 \longrightarrow$ reflection across the y-axis

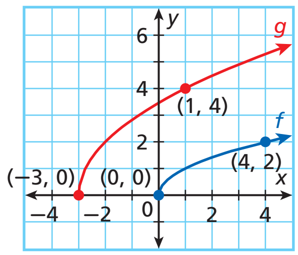
$k \longrightarrow$ vertical translation

EXAMPLE 3 Applying Multiple Transformations

Using the graph of $f(x) = \sqrt{x}$ as a guide, describe the transformation and graph each function.

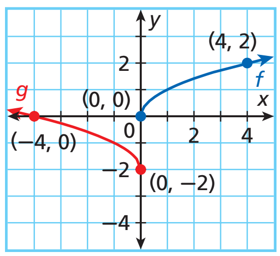
A $g(x) = 2\sqrt{x + 3}$

Stretch f vertically by a factor of 2, and translate it 3 units left.



B $g(x) = \sqrt{-x} - 2$

Reflect f across the y-axis, and translate it 2 units down.



CHECK IT OUT! **Using the graph of $f(x) = \sqrt{x}$ as a guide, describe the transformation and graph each function.**

3a. $g(x) = \sqrt{-x} + 3$ **3b.** $g(x) = -3\sqrt{x} - 1$

EXAMPLE 4 Writing Transformed Square-Root Functions

Use the description to write the square-root function g.

The parent function $f(x) = \sqrt{x}$ is stretched horizontally by a factor of 2, reflected across the y-axis, and translated 3 units left.

Step 1 Identify how each transformation affects the function.

Horizontal stretch by a factor of 2: $|b| = 2$
Reflection across the y-axis: b is negative
} $b = -2$

Translation 3 units left: $h = -3$

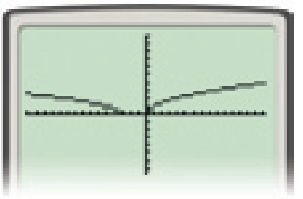
Step 2 Write the transformed function.

$$g(x) = \sqrt{\frac{1}{b}(x - h)}$$

$$g(x) = \sqrt{\frac{1}{-2}\left[x-(-3)\right]}$$ *Substitute −2 for b and −3 for h.*

$$g(x) = \sqrt{-\frac{1}{2}(x + 3)}$$ *Simplify.*

Check Graph both functions on a graphing calculator. The graph of g indicates the given transformations of f.

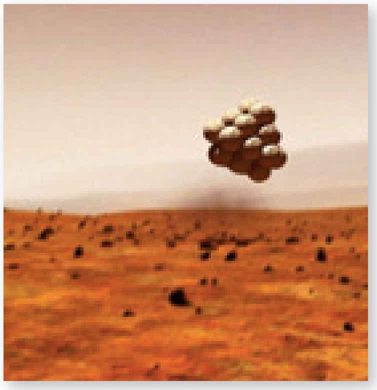


CHECK IT OUT! **Use the description to write the square-root function g.**

4. The parent function $f(x) = \sqrt{x}$ is reflected across the x-axis, stretched vertically by a factor of 2, and translated 1 unit up.

EXAMPLE 5 *Space Exploration Application*

Special airbags are used to protect scientific equipment when a rover lands on the surface of Mars. On Earth, the function $f(x) = \sqrt{64x}$ approximates an object's downward velocity in feet per second as the object hits the ground after bouncing x ft in height.



The corresponding function for Mars is compressed vertically by a factor of about $\frac{3}{5}$. Write the corresponding function g for Mars, and use it to estimate how fast a rover will hit Mars's surface after a bounce of 45 ft in height.

Step 1 To compress f vertically by a factor of $\frac{3}{5}$, multiply f by $\frac{3}{5}$.

$$g(x) = \frac{3}{5}f(x) = \frac{3}{5}\sqrt{64x}$$

Step 2 Find the value of g for a bounce height of 45 ft.

$$g(45) = \frac{3}{5}\sqrt{64(45)} \approx 32$$ *Substitute 45 for x and simplify.*

The rover will hit Mars's surface with a downward velocity of about 32 ft/s at the end of the bounce.

Use the information on the previous page to answer the following.

5. The downward velocity function for the Moon is a horizontal stretch of f by a factor of about $\frac{25}{4}$. Write the velocity function h for the Moon, and use it to estimate the downward velocity of a landing craft at the end of a bounce 50 ft in height.

In addition to graphing radical functions, you can also graph radical inequalities. Use the same procedure you used for graphing linear and quadratic inequalities.

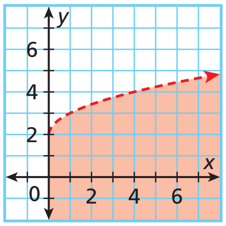
EXAMPLE 6 Graphing Radical Inequalities

Graph the inequality $y < \sqrt{x} + 2$.

Step 1 Use the related equation $y = \sqrt{x} + 2$ to make a table of values.

x	0	1	4	9
y	2	3	4	5

Step 2 Use the table to graph the boundary curve. The inequality sign is <, so use a dashed curve and shade the area below it.



Because the value of x cannot be negative, do not shade left of the y-axis.

Check Choose a point in the solution region, such as $(1, 0)$, and test it in the inequality.

$$y < \sqrt{x} + 2$$
$$0 \overset{?}{<} \sqrt{1} + 2$$
$$0 < 3 \checkmark$$

Graph each inequality.

6a. $y > \sqrt{x + 4}$

6b. $y \geq \sqrt[3]{x - 3}$

THINK AND DISCUSS

1. Explain whether radical functions have asymptotes.

2. Explain how to determine the domain of the function $f(x) = \sqrt{2x + 2}$.

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, give an example of the transformation of the square-root function $f(x) = \sqrt{x}$.

Transformation	Equation	Graph
Vertical translation		
Horizontal translation		
Reflection		
Vertical stretch		